11. Thematic Chapter: Exploratory Software, Exploratory Cultures?

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Abstract. This paper maps out and attempts to explain the common pattern of reaction to exploratory computer-based learning environments. From the point of view of the student and student-software interaction, five common categories of response are described and illustrated by reference to student work with Cabri Géomètre and with Logo. At a more global level, the shaping effect of the school system, termed the inertia cycle, is illustrated by reference to the history of Logo in three countries.

11.1 Background

In this chapter, I sketch what appears to be a common pattern of response to exploratory computer-based learning environments by reference to two pieces of software, Cabri Géomètre[1] and Logo. I will discuss this underlying trend from two perspectives: from the point of view of the student and students' strategies of interaction with the software; and at a more global level taking on broad issues concerned with the adoption of innovatory practices in schools.

In previous papers (Hoyles and Noss, 1992; Hoyles, in press) I looked back over the decade or so of research and development in Logo mathematics in an attempt to draw out implications for the use of software in school mathematics. In the latter paper, my intention was that the reader should look "through Logo" to the broader debate about teaching and learning geometry in school rather than see arguments as rooted in the specifics of any particular software. Despite my exhortations, it is probably the case that some of the points raised were interpreted as "Logo issues" not relevant to evaluation of other software.

[1] I refer here to Cabri as it is the software with which I am most familiar, but the arguments could equally well apply to any other geometrical tool kit or dynamic geometry environment such as Geometer's Sketchpad or The Geometry Inventor.

In the light of extensive experience with other software, I now feel more confident of my analysis. Despite differences between software, I am convinced that parallels can be drawn in terms of: students' use of exploratory software to scaffold their understanding of mathematical ideas, students' interpretations of their computer-based work,and tensions between exploratory learning approaches with computers and the school culture, particularly as exemplified in its evaluation procedures. I will illustrate these common threads by reference to my observations of student work with Logo and with Cabri and by describing the rise and fall of the Logo culture in the past and the developing Cabri culture at the present time.

11.2 Student Response to Exploratory Software Environments

Cabri and Logo are examples of what I would describe as microworlds. A microworld is a setting for activity which presupposes a set of design principles: (1) constituent parts of the world "model" a knowledge domain, and (2) the learning agenda and the strategies used to pursue this are negotiated rather than predetermined. (For an extended discussion of the meanings of microworld, see Hoyles (1993), and Edwards, this volume.) Cabri and Logo are also characterised by continuing development in response to user demand and by their potential for long-term student engagement; they offer possibilities for a multitude of investigations around a range of mathematical topics, including geometry;[2] they have had and continue to have considerable impact on the educational community throughout the world. There are, however, major differences in design between the two pieces of software which might lead one to predict variation in pattern of response from students and schools—a prediction I argue here is largely unfounded.

11.2.1 Different Interfaces, Different Mathematics?

Both Cabri and Logo involve the construction and manipulation of geometrical objects on the computer screen through action and the representation of the action by the computer system. There are, however, differences between them as to the nature of the actions demanded and how these are coordinated: Logo requires their formalisation into a program, which can be viewed as either a summary of the actions performed while building up a screen object or as a representation of the structure and relationships of a geometrical figure; in Cabri, the relationships by

[2] Given this common domain, I will focus my attention here on geometry. Inevitably, therefore, I restrict my attention to an appropriate subset of the tools available—as far as Logo is concerned—to those concerned with turtle geometry.

which geometrical objects are defined are set up by pointing and clicking, and the objects themselves can be picked up and manipulated directly.[3]

A discussion of the implications of these different interfaces leads me into the realm of human-computer interaction, which cannot be treated in any depth here. However, I will raise just one issue. The rationale for a direct manipulation interface is that the user has a sense of engagement with the screen. There is no intermediary between user and system. Thus, the design principle of direct manipulation is that: "The interface should be unobtrusive, not interfering or intruding. If the interface itself is noticed, then it stands in a third-person relationship to the objects of interest, and detracts from the direction of engagement" (Hutchins, Hollan and Norman, 1985, p. 333). But is not this distancing from action exactly what is required for appreciating *mathematical* ideas? The theories of Vygotsky (1986) suggest that some kind of language is needed for the conscious realisation of any activity; action on its own is not sufficient for learning since concepts and their verbal articulation are inextricably linked. This argument has particular potency for mathematics where to some extent the language of expression *is* the mathematics. Language is necessary to plan, discuss, reflect, explain.

This takes us to the conversation metaphor for interfaces, where interaction through a language medium translates user intentions into system instructions. Students have to make the effort to learn how to express themselves in this new medium. But if they achieve this we know from many years of work with Logo that children can exploit the textual language of the system in order to reflect upon what they have done; they use a hybrid language of Logo and natural language as a means of communication (Hoyles, Sutherland and Healy, 1991). What is the language of description and communication in a direct manipulation environment? How do students convey to each other (and to the teacher) what they have done with Cabri? These are, for me at least, open questions and worthy of study.

More generally, one is led to ask what is the interpretation of the geometrical object at the focus of direct manipulations or, in contrast, as described by a computer program? Does the nature of the interface lead students to express their mathematical ideas in different ways? How does this influence the mathematical meanings they develop? Balacheff and Sutherland (1994) in fact suggest that: "both environments [Logo and Cabri] seem to share a domain of phenomenology made

[3]Although Cabri and Logo appear at the present time as examples of software with very different interfaces, interestingly enough this divergence is becoming increasingly blurred. A new version of Logo, Microworlds, incorporates many aspects of direct manipulation. In Cabri, any construction can be built into a macro and named so that a construction can be repeated by simply pointing at the construction name in the menu and clicking on the appropriate initial objects. In later versions of Cabri, it is promised that macros will be constructed from editable linguistic descriptions—more like programs perhaps?

up of drawings at the surface of the screen. But this is a superficial view. If we go more deeply, it appears that there are big differences."

I take another view. Whilst acknowledging the epistemological and cognitive divergence of the two software, the overriding picture for me is one of similarity rather than difference, and it is these convergent elements which I will explore in the rest of this paper. In considering the process of construction of mathematical meaning, I make reference to vignettes of students' work in Logo and Cabri. The Logo work has been described elsewhere (see Hoyles and Sutherland, 1989; Hoyles and Noss, 1992). To illustrate students' work in Cabri, I describe episodes from research conducted in London together with Richard Noss, Lulu Healy and Reinhard Hoelzl, some of which has been written in more detail in Healy, Hoelzl, Hoyles and Noss, (1994), Hoyles, Healy, Noss and Hoelzl (1994), Hoelzl, Healy, Hoyles and Noss (in press).

11.2.2 Tools for Experimentation

To some extent Piagetian theories of learning comprise the underlying rationale for the use of both Logo and Cabri as tools to explore mathematics. Individual action is considered the motor for learning. Students use the software to achieve a goal and in the process they learn by coordinating and reflecting upon the form of their interactions—by developing schemes. Central to this approach is the claim that students must be in a position to construct their own knowledge in situations where it is functional and purposeful; or, put another way, at the core of both environments is the idea of microworld. Additionally, in Logo and Cabri, there is a common emphasis on construction of geometrical figures linked with the intention that by exploring screen models or drawings students come to 'see through' them and apprehend the mathematical characteristics of the underlying geometry (Laborde, 1993).

Thinking of software tools in this way has implications for children's mathematical thinking. The tools come to form part of their cognitive apparatus. As Mason (1992) suggests: "Through exploration, using a device such as compasses, protractor or software, pupils work towards the mental-tool state *in which the device is an extension of their own thinking....*" (my emphasis). Hoyles and Noss (1993) have described a similar phenomenon. We have pointed to how children frequently develop conceptual frameworks, which we describe as situated abstractions, from constructing and articulating mathematical relationships, which are general within a microworld yet are interpretable only by reference to its particular language of description. What we have to decide is the status of these mental tools. Complementary to any situated abstraction and fundamental to the process of its construction is the support system available in the setting, what we termed in the context of computers "the computational scaffolding" (Hoyles and Noss, 1992). This comprises the software tools exploited by the student so as to provide them with the hooks they need on which to hang their developing ideas.

These twin notions of situated abstraction and computational scaffolding are central to our thinking about exploratory software. The point I want to convey here is that frequently children can and do use software to achieve their goals by circumventing part of the mathematical analysis that we might expect them to apply. This allows them first to concentrate on solving a problem, after which they might feel more able to debug the solution procedure, make it more general or "mathematically correct."[4] We have illustrated this phenomenon with numerous observations in Logo settings (see Hoyles and Noss, 1992). For example, a program for a specific geometrical shape might be built, not for its own sake but rather with the intention that it will serve as a skeletal generic example from which generalisations can be made by the substitution of variables for fixed quantities. I give here a brief vignette to illustrate how children's work with Cabri follows a very similar pattern.

Reflecting on reflection in Cabri
In their third session with Cabri, after some exploration with the creation and construction tools available, we asked our group to reflect a flag in a mirror (see Figure 1). The children had not been taught the relevant ruler and compass construction and had to develop a "Cabri method."

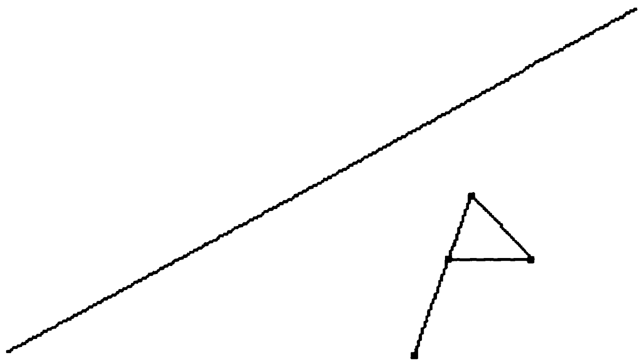

Figure 1. The reflection task.

Here I describe how Helen and Katie found a way to solve the problem by first simplifying it and then using the medium as a platform from which to construct a solution. The two girls first dragged the mirror line to *look* vertical keeping the flags position unchanged—something you cannot do with paper and pencil. Then they drew a line segment through one of the points on the flag, P, and a basic point, Q, on the other side of the mirror making PQ *look* horizontal. They then dragged Q so that it *looked* the same distance

[4]diSessa (this volume) mentions a similar phenomenon which he calls, "learning by cheating" where children mess up the underlying program of a microworld but nonetheless learn by giving themselves a concrete hypothetical to think about.

from the mirror line as P (see Figure 2). They had a "solution," something on the screen on which to reflect, which turned out to be crucial scaffolding for solving the problem more generally.

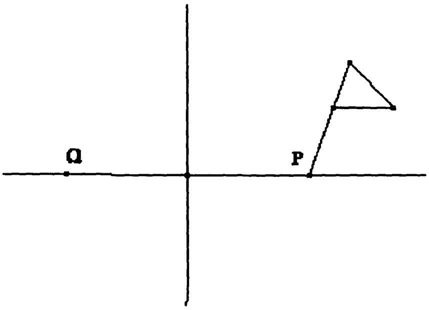

Figure 2. Constructing a scaffold.

Despite having "found" the image of P in the mirror line, Katie was quite aware that this would not do; nothing was constructed. After some exploration, the girls came up with the idea that a circle could measure the equal distances from the mirror, so they constructed a circle with the mirror intersection point, O, as centre and the radius, OP and found the image of P by the intersection of this circle with their line (see Figure 3). They did not bother to check by measuring and were simply convinced of the correctness of their strategy, immediately starting to repeat the process for different points. The pair had found a way to reflect any point—but their strategy relied on the first line being drawn as horizontal by eye. Or put another way *they had solved the problem for distances but not angles.* We have many examples of children simplifying a problem so that they could cope with it by dealing with one variable at a time. But with Cabri, this simplification need not just solve one special case but rather can provide a route to a general solution as shown in this case.

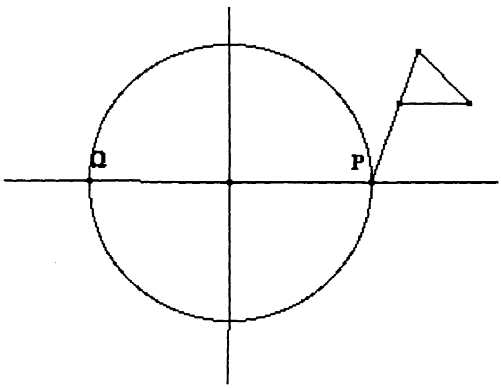

Figure 3. Using a circle to measure lengths.

We asked the two girls what would happen if the first line, PQ, going through the mirror, was moved. They both knew that this would "mess up."[5] Once they had noticed this—and because they had in front of them what they wanted to produce—they saw that they needed to construct a perpendicular line from P to the mirror. They did this and repeated their circle construction—with the mirror line still vertical. It is at this point that the medium really came into its own. The mirror was dragged back to its original position taking with it the constructed image point Q as shown in Figure 4.

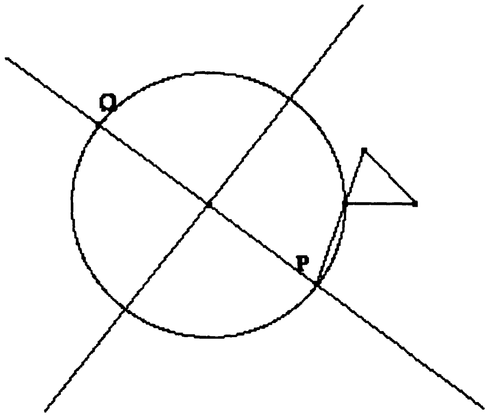

Figure 4. Returning the mirror to its original position.

This example illustrates how children can use exploratory software to come up with solution strategies that do not conform to any recognised routine. The software is used to *negotiate* a problem solution, which is unique to the setting—in this case drawing a solution by eye, then dealing with one constraint at a time. With other software the nature of the scaffolding is different. In Logo it might be a procedure "template" which provides the form of a mathematical structure but with the program syntax and other details to be determined. The point is that the software provides students with the opportunity to build *their own* mathematical constructions. Whatever the software is, one aspect of the teachers' role must be to help students to use the software in ways *they* choose—resisting any temptation to impose the usual well-trodden paths.

[5] In the first sessions of Cabri work, we had devised a way to help children come to see that properties that are to be preserved need to be constructed. We came up with the idea of 'messing up'—after a figure was drawn it could be dragged (by anybody including us!) to see if it became unrecognisable—that is whether the different objects within the design moved together in a sensible way or not. Messing-up gave us a language in common with our pupils, and afforded us all a mutually acceptable mode of validation for constructions.

11.2.3 Centration on the Screen

There are always two sides to any coin! As far as exploratory software is concerned one side is the facilitation of student-generated pathways to mathematical knowledge: but, simultaneously, the other is the inevitability that students might not accept or even notice the educators' agenda. Students can become centrated on the screen and see their task as one of making the pictures on the screen "look alright" as opposed to ensuring that the mathematical structures are in place. This is a complex question because there are some aspects of screen feedback that need to be ignored from a mathematical point of view—such as the "wrinkles" arising from poor screen resolution—while there are others that need to be attended to. How do students know what to notice and what to ignore? Here one can discern an inevitable paradox endemic to learning with exploratory software, which we have termed the play paradox (Hoyles and Noss, 1992). How can children know what to attend to if they do not already know the mathematical area under investigation? In the context of graphing software, Goldenberg (1991) points to the same dilemma: "algebraic sophistication can lead us to ignore appearances" (p. 143) but without this background, inductive reasoning based on a mixture of perception, mathematical knowledge and even guesswork, all mediated by the software, can lead to weird and wonderful generalisations!

Thus we expect children to solve a problem by transcending the feedback from the screen output and the visual cues available, drawing instead upon their mathematical knowledge. Research from the Logo world tells us that this rarely happens without teacher intervention (Hillel, Kieran and Gurtner, 1989; Hoyles, Noss and Sutherland, 1989). Once again we can observe the same phenomenon of screen centration when students work with Cabri, as illustrated in the following short vignette:

> *When is a rectangle, really a rectangle?*
> In their third session, Billie and Natalie were asked if they could construct a rectangle using Cabri.
>
> Despite our emphasis in previous sessions on the importance of construction in ways that could not be messed up, Billie and Natalie, much to our surprise, brought the grid up on to the screen and used this to place carefully four points to produce a rectangle. They were delighted with their achievement, believing that they had answered the question!

The kind of behaviour described above forces us to problematise what is "wrong" with the children's productions with exploratory software. In the vignette, the girls used their knowledge of the parallelism and perpendicularity of lines on a grid to trace out a rectangle—so the drawing *did* contain two sets of parallel and perpendicular lines based on the grid. They had made a shape which satisfied in some way the conditions of the required geometrical object but had not used the criteria we expected. The lines were simply not Cabri constructions.

11.2.4 Preference for the Direct Mode

In contrast to software that "dictates" a sequence of steps to a solution, exploratory software, by definition, allows students to adopt a variety of problem-solving strategies—from trial and error to the highly sophisticated. But researchers have noted that children frequently develop a sophisticated approach to an investigation with exploratory software, even to the extent of constructing tools that will do some of the elementary work for them, only to ignore these tools later and revert to simple, direct approaches with immediate feedback. This has been widely documented in the Logo literature—for example, students build procedures and then do not use them (see, for example, Hoyles and Sutherland, 1989; Hoyles and Noss, 1992).

Again I have observed a similar phenomenon in a Cabri environment where children have struggled to construct macros only to ignore them later, as illustrated in the following vignette:

> *Missing the Macro*
> In our fourth Cabri session, we introduced the children to macro constructions in Cabri. All the students managed to construct simple macros for a variety of shapes, equilateral triangles, intersecting circles, etc. They then set about making more complex patterns which incorporated repeated constructions. We felt this would provoke them to see the need for a macro. Billie and Natalie wanted to draw the pattern of parallelograms illustrated in Figure 5. They decided straight away to make a parallelogram macro and apply it over and over again. After tremendous efforts (and some help) they managed to construct a macro with three initial points, and eventually, after several false starts due to mistakes in orientation, applied this repeatedly to draw one line of parallelograms.
>
> They had drawn the top line in Figure 5, so then had to work out how to draw the line below.

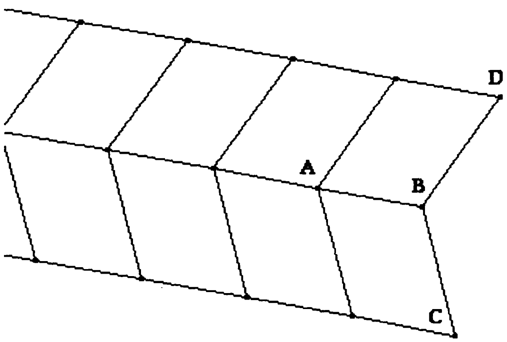

Figure 5. Billie and Natalie's parallelogram pattern.

> Two of the points, labelled A and B in the figure[6] were already constructed by the 'top' line. The girls now had to find a way to construct a third point, labelled C, so they could use their macro. After some discussion, they noticed that the point C was symmetrical to D in the line through AB. So C was constructed with great relief all round. But then (much to my surprise), the girls did *not* use their macro as planned (with initial points A, B and C) but simply 'reverted' to the primitive menu item of parallel lines.

Why do students choose not to apply the general strategies which we know are available to them? Is it that they are not confident with their more sophisticated tools, whether they be Logo programs or Cabri macros? In the Cabri example, it was in fact clear that the two girls were never sure which way the parallelogram would be drawn after the three initial points had been clicked so there was a problem of order and orientation which was never overcome. Or possibly the obstacle is emotional rather than cognitive—students do not feel "ownership" of either the macro or the procedure? Or it may be that even though in our view the task encouraged the use of more general tools—in the vignette, a great many parallelograms had to be constructed so it would be efficient to use the software to build a macro for one which could then be repeated—this view may not be shared by the students. Or are we really facing here an issue of school culture, where the thing to do is to choose the approach with the least cognitive demand, one that is efficient in simply getting the job done in a step-by-step routine manner?

11.2.5 Exploring the Software or the Mathematics?

Although exploratory software environments are open, they inevitably have constraints and the nature of these might not be appreciated by the user. How can an inexperienced user *know about* these software constraints? How can they differentiate between what might in principle be correct but simply not allowable in a particular environment? How can children discriminate between software effects and effects which characterise mathematical structures and relationships? In Logo, confusion may arise, for example, from failure to comprehend how values of variables are passed from a Logo procedure to its subprocedures (see Hoyles and Sutherland, 1987). Again we find similar situations with Cabri: We have noted that students thought that dragging a shape changed it in a proportional way—an artifact of how a point constructed to lie anywhere on an object happens to move with the object. How can students distinguish fundamental characteristics of geometry from such features of Cabri's design? Another potential source of confusion in Cabri is that any hierarchy of relationships which has been established cannot be modified. This is illustrated in the following vignette:

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[6]All the labels are added by the author to help clarify the story. They were not part of the students' work.

Relative Relationships?
Cleo and Musha were trying to construct a rectangle, but this turned out to be a quadrilateral, ABCD, with two right angles and one pair of parallel sides (illustrated in Figure 6). The buggy side, AD, needed fixing as it was not perpendicular to AB or DC and certainly not parallel to BC. The girls had constructed the line DX perpendicular to CD, but clearly this did not pass through A. After much thought, Cleo began to drag A and said, "We've got to make A a point on object." We assume Cleo meant that A had to be made to lie on the perpendicular DX, while keeping all the previously constructed relationships intact.

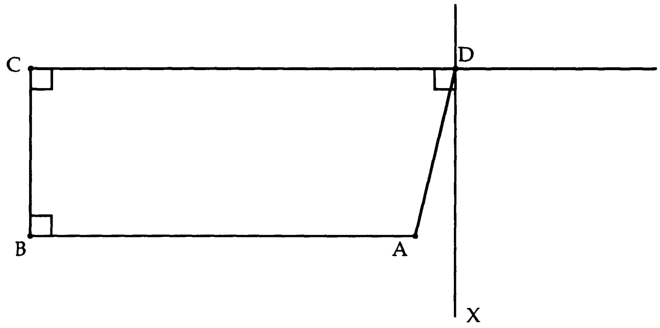

Figure 6. Constructing a rectangle.

A could be moved to DX by eye, but Cleo wanted to change the status of A, a basic point, to be a point on DX. This seems quite reasonable and if it could be done would make a rectangle. But Cabri does not allow this. A hierarchy of dependencies had been set up—C depends on A and B, D depends on C—so Cabri does not allow A, the starting point, to depend on D. An episode reporting a similar phenomenon is described in Hoelzl (1993).

There is clearly a need for some tuition about the mechanisms of exploratory software when confusions such as those mentioned arise. Sometimes software just does not let you model the situation as you see it even if your model is quite legitimate from the point of view of mathematics! Discussion of these incidents should be beneficial for understanding the mathematics and becoming familiar with the foibles of the software.

11.2.6 Student Meanings and Our Meanings

There is a huge literature describing the alternative conceptions students hold of mathematical ideas; how the meanings they construct frequently differ from those we expect. My contention is that any divergence in meaning is brought under a spotlight when students are using tools more sophisticated than a pencil.

It is clear that we cannot ignore how mathematical ideas are shaped by the tools available, the way they constrain actions and representations. This is true for paper and pencil technology just as much as computers. It is also true that despite our constructivist attempts to work with children's perspectives we frequently do not notice a student's viewpoint. This gives another twist to the contentious issue of the visible or the invisible interface. Tools should always have a visible face—we should understand them, how they work and even how they might be modifiable. Yet this visibility should not be confused with the visibility of the products of interaction with the tools. As Lave and Wenger (1991) argue, "Invisibility of mediating technologies is necessary for allowing focus on, and thus supporting visibility of, the subject matter. Conversely, visibility of the significance of the technology is necessary for allowing its unproblematic—invisible use!" (p. 103).

My contention is that the interpretations which emerge while working in a new computer environment, *because of its unfamiliarity,* "stop us in our tracks" and force us to notice student conceptions. Thus the fact that the software constrains children's actions in novel ways can have rather positive consequences for constructivist teaching. *The visibility of the software affords a window on to the way students build conceptions of subject matter.*

We have noted this phenomena when children have been working with Logo. For example, when two girls were drawing a solar system of stars, Mary asked if they could design a large star and then "shrink" it. We helped her to write a procedure for a star with a scale factor operating on lengths as input. The girls tried to make sense of the input by trying different values during which process one aspect of Mary's conception of decimals which had not previously been picked up by the teacher became apparent. Mary had discovered that multiplying distances by 0.5 would make the star smaller. She then predicted that multiplying by 1.5 would also make it smaller because "1.5 is not a whole number." She typed `STAR 1.5` and was of course surprised at the output—an even larger star! After further experimentation, she was able to work out the significance of the numbers before and after a decimal point. (This incident has also been reported in Hoyles and Sutherland, 1989.)

How meaning can be illuminated in this way is illustrated in the following vignette taken from our Cabri work.

> *Interrogating Intersection*
> During our work with Cabri we became exercised as to why the children seemed to find the construction of intersection points so difficult.[7] We decided that there were several obstacles to appropriating this idea. First there was a language problem—intersection turned out to be an unfamiliar word,

[7]In Cabri, the intersection point (or set of points) of two objects has to be constructed and in the version of Cabri we were using this construction is effected by opening the construction menu at intersection and then clicking on the appropriate two objects.

at least in its written form, so the students were unable to draw on any spontaneous intuitions as to what the menu item might mean. We therefore tried to introduce the word into the classroom vocabulary at every opportunity. But this did not solve the problem! Next, we became aware that the children did not naturally think of constructing intersection points. This is hardly surprising: In real situations, indeed in any non-mathematical situation, an intersection is completely defined by the mere act of crossing two objects.[8]

But there turned out to be yet another obstacle which took us completely by surprise. In a class session, we drew on the blackboard a circle crossed by a line and asked the children to identify any points of intersection. Clearly there were difficulties.

Nobody volunteered. Suddenly one of us (Richard Noss) came up with a conjecture as to what was the problem. He suggested that the students' meaning for circle might be the surface of a circle rather than its circumference—after all one frequently sees in text books a picture of a disk and a label "This is circle." It was plausible, therefore, to conjecture that the children might be thinking of the disk, not its boundary, which would clearly give rise to problems as to the position of intersection points. Further investigation suggested that this indeed was the case and their meaning of circle was at variance to ours and to that of their teacher.

The vignette nicely captures an all too common situation whereby unbeknown to teachers, students, perhaps over many years, conceive of mathematics rather differently from them. Changing the setting poses new problems and, in pursuit of their solution, helps us to confront our assumptions that students see the mathematical world as we do.

11.2.7 Where is Meaning Constructed?

In the preceding sections 2.2–2.6, I have focused on the paradoxes endemic to student/software interactions: How exploratory software can provide scaffolding for building mathematical meaning yet can also be used to bypass mathematical analysis; how, on the one hand, students construct meanings that can be traced to the idiosyncrasies of software design rather than the characteristics of the mathematics; but, on the other, it is just because the software demands an approach which is novel that its use throws light on student meanings. There is, however, another tension arising from the use of exploratory software and classroom norms. This tension is not unrelated to the paradoxes mentioned earlier since classroom culture and student interaction are mutually influential. However, it demands a more general level of analysis, where explanations are sought at the level of the system rather

[8] In the latest versions of Cabri we understand you will simply have to click on an intersection point.

than in the activity structures of the children. I now turn to consider this more global perspective and discuss the place of exploratory software in school. My claim is that any consequences of differences in software design pale into insignificance when set against the shaping effects of the school system—the playing out of what I will term the inertia cycle.

11.3 The Inertia Cycle

In this section, I describe what I see as an almost inevitable pattern of response to innovation, the inertia cycle. This cycle begins with a surge of excitement, after which the innovation is either dropped or is smothered while the system wobbles back to its previous equilibrium state. Over the past decade or so, every "good" innovatory software for mathematics has been greeted with a mixture of excitement and apprehension. Amongst the enthusiasts, there is joy at the possibility of new approaches to mathematics, excitement in the face of the new challenge to the straight jacket of the curriculum. But we look a decade later and the software has either disappeared or has been institutionalised in ways which make it almost unrecognisable. Discussions centre not on mathematical ideas and investigations but on "curriculum fit." To take just a few examples in the U.K.: In the late 70s, BASIC was dominant, accompanied by claims for the importance of writing algorithms; in conferences in the mid 80s, there was talk about the potential of "mathematical programming" (a compromise formula to allow discussion of Logo without actually having to name the language!); then came Logo itself followed by spreadsheets. Each piece of software has suffered a similar fate. Now, in the early 90s, Cabri and computer algebra systems are "the flavour of the year." What is in store for them?

To characterise the ebb and flow of the inertia cycle, I will trace the story of Logo. The story is complex not least because the way the inertia cycle is enacted varies between countries. So to illustrate it in its different guises, I point to highlights of the Logo story as it was transacted in the U.K., U.S.A. and Germany. For example, in the U.K., arguments were mainly amongst curriculum developers and policy makers, while, by way of contrast, in Germany, the battle raged amongst academics. The nature of the debate and the evidence called upon varied but in each case the cycle was the same—a new vision was put forward only to be crushed in the return to the status quo.

11.3.1 The Logo Story in Three Countries

In the U.K., there was a surge of interest in Logo in the early mid-80s. There were large conferences which provided a forum for participants from all parts of the educational community to meet. There was enthusiastic curriculum development together with some research. Despite problems with access to hardware, excitement spread into classrooms. But since the late 1980s, there has been a marked tapering off of interest. There is a widespread feeling that Logo is "over"! The software does not feature in major conferences; there are no new research initiatives. Logo is in fact still around, but in what form? A glance at any curriculum document, shows that Logo has been reduced to sets of turtle graphics commands (DES, 1991); it exists merely within carefully designed sequences of exercises aimed at well-defined sets of skills. Logo has suffered the fate of institutionalisation, incorporation into the curriculum.

Interestingly enough, one consequence of this transformation is that many mathematics teachers regard Logo use in their classrooms as unproblematic—it is an everyday part of their toolkit in much the same way as a calculator or a piece of chalk. This effect of institutionalisation is illustrated in the following extracts from interviews with some mathematics teachers in a London secondary school talking about Logo and justifying its use in their classrooms:[9]

> "I mean I really like Logo and I think its very good for getting them to think about angles and I mean writing programs...I mean logically structuring...."
> "A lot of the reason for using Logo is because it's a very open environment to use maths in and it develops a lot of logical skills and...reflective skills; I think reflecting on where you've gone wrong...students are much more likely to go back and correct their own mistakes rather than depending on somebody else to correct their mistake....I mean that Logo is very different to the others because it's very, very powerful and it's very powerful for doing something that is your own idea as well as for reproducing something else...."

Alongside this acceptance of the cognitive benefits of Logo is the claim that Logo is a world into which students can enter without too much intervention, too much hassle for the teacher—something which is contrasted with the newly-imported Cabri:

> "I mean Logo is so much more intuitively obvious [than Cabri]...."
> "With Logo you can in the first session just start them off and see what happens...."
> "As a starter I find Logo quite nice...."
> "I find that Cabri (and to some extent with spreadsheets)...students are taking a time to get used to it...it's quite demanding in terms of needing continuous time... and you actually have to suspend your helping of other students...."
> "If they have a problem (with Cabri) they need my instant help...."

[9] These teachers were working with Hoyles, Noss and Healy as part of a project on the use of portable computers in schools.

So with familiarity, Logo has become constrained, neutralised, even easy.[10] How justified is this? Logo, through drawing with turtles, certainly connects with what children want to do and can do. It is easy to get going. This comfortable situation can soon throw up serious mathematical challenges—something positive, provided the system allows them to be pursued and provides some support!

In the U.S. we find a rather different trajectory for Logo. After tremendous acclaim for Logo in the early 80s, with Logo used in most parts of the curriculum, there has been a relentless backlash. Little is written about Logo now, and, indeed, my impression is that teachers and developers even apologise for using Logo or hide their Logo use. A consequence of this swing of the pendulum has been the isolation of "Logo ideas"—they are interpreted as simply of relevance to "Logo people" and not to the wider educational community. My evidence for this claim is largely anecdotal but, browsing through mathematics education journals, I am struck by the absence of reference to findings from work with Logo when used as a tool to investigate mathematical conceptions or as a source of insight into the exploratory learning of mathematics.[11]

But even in the U.S., Logo has not disappeared! Logo has been reborn as Microworlds with an new interface which incorporates many aspects of direct manipulation (as mentioned earlier)—turtles can be dragged to change their position and heading; and of graphical packages—there is a drawing centre, a flip tool and a paint bucket. Quite complex projects can in fact be undertaken with little or no programming. So, from one perspective Microworlds has "diluted" Logo, some would argue to the point of its extinction. Microworlds is targeted for the non-mathematical audience—for a culture of drawing, games and home computing. This has the advantage of connection with the "kid culture" of today, but our experience indicates that mathematics cannot be learnt in quite this way. It is not a spontaneous product of everyday activity. Mathematics is in fact one focus for Microworlds; there is Microworlds Math Links (1993), part of which comprises a range of projects and investigations. Yet, here again, you can discern the process of institutionalisation—the last stage of the inertia cycle. The Teacher's Resource book states: "Microworlds Math Links *was designed to support* the NCTM standards" (p. 5, my emphasis). It is clear that the agenda is set up by the system rather than by the student or teacher!

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[10] Articles about Logo in the classroom appear regularly (see for example the U.K. journal for teachers *MicroMath,* 1994). This is not, of course, the complete picture. Some teachers still reject Logo while others still find an approach to Logo which challenges routines and resonates with their aims and objectives.

[11] One incentive for writing this paper was to try to break down this isolation by drawing out the similarities between Logo and Cabri. I remember clearly a few years ago when I presented a paper on geometry which happened to incorporate the use of Logo that the response from some of the audience was that my work was interesting but really only of relevance to Logo researchers rather than the mathematics education community more generally!

In Germany, we find yet another exemplification of the cycle of inertia in that all the phases were transacted at the level of theory, with little or no reference to practice. Logo was praised in the pages of books and journals and in academic discussions and then rejected in the same forum.[12] When *Mindstorms* was published in Germany in 1982, in his lengthy foreword, Otte hailed it as one of the best books on mathematics didactics. A long review of *Mindstorms* written by Hans Niele Jahnke from University of Bielefeld appeared in *Educational Studies in Mathematics* in 1983. It concluded: "This book should be read by everybody concerned with teaching mathematics. It is exceptional because of its broad views. The efforts at reforming the mathematics curriculum in the last 20 years have shown that no progress can be achieved without such general perspectives. Beyond that, personal involvement makes this book a reading matter which is very inspiring and provokes the reader to reconsider his own fundamental views" (Jahnke, 1983, p. 100). Other books and some research followed which sowed the seeds for the academic debate, part of which reached the pages of the *Journal für Mathematik* and finally led to a hundred page critical article by Professor Bender in June 1987 (Bender, 1987).

The first question which immediately comes to mind is whether the publication of such lengthy critiques was normal practice for this journal? Apparently not—a brief survey suggests that articles were normally about 20-30 pages long. Why was an exception made in this case? Of course one can never know. But it seems reasonable to surmise that the editors were disquieted by Papert's ideas and in sympathy with Bender's position. Additionally, as far as I understand it, Bender's article was not only distinguished by its length but also by its style and tone—it matched Papert's rhetorical language by including polemic and irony not usually found in academic papers. Reactions to Bender were immediate in the form of two replies in the next issue of the journal, but Bender's article proved to be the more effective and was frequently quoted in later debates. In fact after its publication there was a deafening silence in Germany concerning the question of Logo. The debate on computer use in mathematics continued, but Logo's fate was sealed.

11.3.2 How has Logo Been Evaluated?

So Logo has passed through the inertia cycle. It has disappeared or has been incorporated into the curriculum, at least as far as the three countries described are concerned. In the process it has lost much if not all of its potential as an exploratory environment. Can we be more precise as to why this happened? My view is that classrooms as they are at present constituted sit unhappily with exploratory environments. This point, however, is rarely made. Instead, evaluation has tended to focus on Logo itself rather than on the environment into which it has been inserted. The question at the heart of the debate about Logo's future in many countries has

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[12] I am indebted to Reinhard Hoelzl for his comments on the discussions of Logo in Germany and for his translation of excerpts of Bender's paper.

been in Papert's terms, technocentric: "Did *Logo* work?" On the whole the answer has been "no," which has given momentum to Logo's downward spiral. Yet, what was the evidence?

Many evaluation studies were undertaken with a traditional pre and post-test methodology. Perhaps rather surprisingly these show a largely positive picture of Logo "effects" (De Corte, 1993). Yet, the findings of the first evaluation study undertaken in this paradigm by Pea and Kurland (1984) seems to have had an overwhelming influence, particularly in the U.S. Pea and Kurland found that children did not learn much Logo in a noninterventionist setting and also that the expected improvement in planning skills was not evident from the tests results. Why have these results been interpreted as a devastating evaluation of Logo rather than of a particular type of learning environment? How has it been accepted that written tests can serve as adequate assessment of learning in such complex situations?

In fact psychologists still use Logo, (well the turtle), as a site for their experimental work using a similar methodology. Simmons and Cope as late as 1993 report some experimental work with a pre, post-test design, which concluded that the turtle environment encouraged the adoption of trial and error approaches to the detriment of the mobilisation of conceptual knowledge and gave evidence of children's failure to appreciate rotation and angle in Logo. But what is the basis of their claims? The children tested had little or no experience with Logo. They were taken to a laboratory, given a brief introduction to FD, RT, LT and CS (apparently not BK!), allowed ten minutes to "experiment" and then given 6 angle/rotation problems "presented by way of a Logo microworld" and 6 "similar" problems on paper. What sense could the children have made of the problems presented to them? How could Logo have any meaning after so little experience with the software?[13] How is it possible to assume that conceptual learning can be a measurable outcome of a series of tasks over such a short period of time?

What is fundamental in helping us to think about evaluation of exploratory software in general is to examine the learning theory underlying this study and, indeed, that of Pea and Kurland and compare it with that of "microworld learning." It is very clear, for example, that this study puts children into a world far removed from any exploratory vision for Logo use. We therefore have to face the question as to how far this type of experiment can provide any sort of evaluation of learning in a microworld, which brings me to the next section.

[13] I noticed an almost incidental but for me a telling remark in the Simmons and Cope study: "One pair of children's results were excluded since it became apparent that after 40 minutes neither child was sure which way the turtle was pointing" (Simmons and Cope, 1993, p. 166).

11.4 Evaluating Exploratory Software

What do success or failure as measured by written tests offer as a basis for judgements of the efficacy of exploratory software? Given the divergence of goal, and theoretical framework, my feeling is rather little. Yet given there are few alternative modes of evaluation available, this traditional approach based on paper and pencil testing is widely in evidence. This overvalues the paper and pencil medium to the detriment of the computer and assumes that responses to questions which atomise mathematical understanding into isolated packages is evidence of learning. The question of devising alternative means of evaluation is pressing. Sadly, all the energy put into designing exploratory software and associated activities is not matched by attention to alternative modes of assessment.

We do not have accepted and acceptable methods of evaluation which are authentic in terms of the aims of exploratory computer-based learning environments?[14] For example, assessment should be based on tasks which make sense to students and which do not necessarily embrace a right/wrong epistemology; assessment should encompass a range of possible student responses including responses made on a computer; assessment should take into account long-term development and possible attitudinal influences. It may be that the design of this new form of assessment should "anticipate" a skewed rather than a normal distribution of student responses for any particular set of tasks; that is only a small proportion of students are expected to succeed. Naturally, this course of action must assume that all students can find at least one niche in which to flourish. Such an approach would respect pluralism in assessment as much as pluralism in activity; it would aim for deep involvement in selected areas rather than average performance across a wide range. It would of course require a radical rethink of what is regarded as progress and fulfillment in school.

11.5 Concluding Remarks

So what lessons can we learn from the Logo experience, and will Cabri suffer the same fate? It is too early to identify the sources of resistance to Cabri and how the debate will be played out. But, history indicates that even if not dropped, Cabri will change as the cycle of inertia slowly turns. The clues are already there. I noted the publicity for Cabri by Brooks/Cole and present two (edited) quotations from U.S. university teachers:

> "I love the ease of creating, changing, and measuring given figures...This system is far easier to use than MacDraw or the Geometric Supposer...."
> "I'm impressed with the program's user friendliness and potential to serve as a supplement to instructional materials for teaching geometry."

[14] I am indebted to Mike Eisenberg who led a discussion after my talk at the NATO workshop and raised the issue of authentic evaluation.

First note the emphasis on "easiness". But Cabri (like Logo) is *not* easy, although the sources of difficulty might not be the same. Cabri's success depends on the existence of a geometrical culture; one which prioritises and rewards conjecturing and hypothesis testing. In the U.K., where this culture is weak, Cabri has been described as "hard"—students need considerable mathematical sense to look for variants and invariants rooted in geometry. Perhaps it is this mathematical complexity which will be the seeds of Cabri's future demise, at least in the U.K.? But again, note also the reference to "curriculum fit" in the second quotation. Do we see here the neutralising of exploration—another enactment of the inertia cycle? Finally note the failure to appreciate what Cabri is all about in the suppression of its geometrical purpose by the comparison with MacDraw.

Some might argue that Cabri is less likely to disappear because it is more embedded in the existing curriculum. But embedding in the curriculum is not an absolute concept—it depends on habit and what teachers have grown used to—reference the comments of the U.K. teachers earlier. Even if embedding is achieved, students may not necessarily explore in the ways anticipated—they take short cuts from a mathematical point of view and use other means for solution as illustrated in my vignettes. Additionally, as I have argued, we have to find ways to evaluate Cabri that treat it seriously.

The point is that exploratory software is *not* simple and its use is not simple. It is easily misrepresented and its purpose easily abused by inadequate conceptualisation leading to crude means of evaluation. Understanding and facing up to this complexity is the only way out of the cycle of inertia; the only chance of realising the potential of exploratory software. This is the challenge we need to face.

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